FUNDAMENTAL CONCEPT

Nominal vs. Effective Interest

Nominal Rate (r)

The **nominal interest rate** is the stated annual rate without considering compounding frequency.

Characteristics:

- ✓ Stated in loan documents
- ✓ Does NOT reflect true cost
- ✓ Used for marketing purposes

Example: "12% per year compounded monthly"

Effective Rate (i)

The **effective interest rate** is the actual rate earned or paid when compounding is considered.

Characteristics:

- ✓ Reflects true cost of borrowing
- ✓ Always ⩾ nominal rate
- ✓ Used for accurate comparisons

Example: 12% compounded monthly = 12.68% effective

Why the Difference Matters

Comparing Loans

Two loans with the same nominal rate but different compounding periods have different effective rates.

Investment Returns

More frequent compounding yields higher returns. Daily compounding > monthly > annually.

Consumer Protection

Laws require disclosure of effective rates (APR) to protect consumers from misleading nominal rates.

CALCULATIONS

Effective Annual Interest Rate

The Formula

$$i_a = (1 + r/m)^m - 1$$

i_a = Effective annual interest rate

r = Nominal annual interest rate

m = Number of compounding periods per year

Compounding Periods

Compounding	m (periods/year)
Annual	1
Semiannual	2
Quarterly	4
Monthly	12
Daily	365

Example Calculation

Problem:

Calculate the effective annual rate for 12% compounded monthly.

Solution:

Given: r = 12% = 0.12, m = 12

Formula: $i_a = (1 + r/m)^m - 1$

Calculation: $i_a = (1 + 0.12/12)^{12} - 1$

$i_a = (1.01)^{12} - 1$

$i_a = 1.1268 - 1 = 0.1268$

i_a = 12.68%

Key Insight: 12% nominal = 12.68% effective

Part 02

Annual Learning Overview

01. Effect of Compounding Frequency

Comparison Table

Effective rates for 12% nominal interest:

Compounding	m	Effective Rate
Annual	1	**12.00%**
Semiannual	2	**12.36%**
Quarterly	4	**12.55%**
Monthly	12	**12.68%**
Daily	365	**12.75%**
Continuous	∞	**12.75%**



Key Observations

↑ More Frequent = Higher Effective Rate

As compounding frequency increases, the effective rate increases but at a decreasing rate.

∞ Limit: Continuous Compounding

Daily and continuous compounding produce nearly identical results. The maximum effective rate is achieved with continuous compounding.

Practical Implications

For most practical purposes, monthly compounding is sufficient. Daily compounding offers minimal additional benefit.

02. Continuous Compounding

The Concept

Continuous compounding occurs when interest is compounded infinitely often - at every instant. This represents the theoretical limit of compounding frequency.

$$i_a = e^r - 1$$

i_a = Effective annual interest rate

r = Nominal annual rate

e = Euler's number ≈ 2.71828

When is it Used?

Theoretical analysis: Mathematical models and derivations

Financial derivatives: Option pricing models (Black-Scholes)

Academic research: Simplifies complex formulas

Example Calculation

Problem:

Calculate the effective annual rate for 12% with continuous compounding.

Solution:

Given: r = 12% = 0.12

Formula: $i_a = e^r - 1$

Calculation: $i_a = e^{0 \cdot 12} - 1$

$i_a = 1.1275 - 1 = 0.1275$

i_a = 12.75%

Compare: Monthly = 12.68%, Continuous = 12.75%

Part 03

Effective Rate for Any Time Period
&
Practice Problems

03. Formula

$$i = (1 + r/m)^c - 1$$

i= effective rate for the period
r = Nominal annnual rate
m = Number of compounding subperiods in the time period
c=Number of comp

Problem:

Nominal annual rate = 10%, compounded daily (365 days per year).

Find the effective rate for 30 days.

Solution:

Given

r = 10% = 0.10

m = 365

c = 30

$$i=\left(1+\frac{0.10}{365}\right)^{30}-1=0.8252\%$$

02.

Practice Problems

1. Effective Annual Rate

Question: Calculate the effective annual interest rate for a nominal rate of 10% compounded:
a) Semiannually b) Quarterly c) Monthly d) Daily
Formula:
$i_a = (1 + \frac{r}{m})^m - 1$
r=0.10

a) Semiannually (m=2)
$i_a = (1 + \frac{r}{m})^m - 1 = 10.25\%$

b) Quarterly (m=4)
$i_a = (1 + \frac{r}{m})^m - 1 = 10.38\%$

c) Monthly (m=12)
$i_a = (1 + \frac{r}{m})^m - 1 = 10.47\%$
d) Daily (m=365)

$$i_a = (1 + \frac{r}{m})^m - 1 = 10.52\%$$

2. Comparing Credit Cards

Question:
Credit Card A: 18% compounded monthly
Credit Card B: 18.5% compounded quarterly
Which has the lower effective annual rate?

Step 1: Calculate EAR for Card A

$$i_{a,A} = (1 + \frac{0.18}{12})^{12} - 1 = 19.56\%$$
Step 2: Calculate EAR for Card
$i_{a,B} = (1 + \frac{0.18}{12})^{12} - 1 = 19.82\%$

Conclusion

boldsymbolCardA has the lower effective annual rate and is the better deal.

3.Finding Nominal Rate

Question: Find the nominal rate that yields an effective annual rate of 10% when compounded quarterly.
Rearranged Formula:

$r = m[(1 + i_a)^{\frac{1}{m}} - 1]$

i_a=0.10,m=4

$r = = 4*[(1 + 0.10)^{\frac{1}{4}} - 1] = 9.65\%$

4. Future Value Comparison

Question: Compare the future value of $5,000 after 5 years at 10% compounded:
a) Annually b) Monthly c) Continuously
Formulas:

- Discrete compounding: $FV = PV(1 + \frac{r}{m})^{mt}$
- Continuous compounding: $FV = PV*e^{rt}$

PV=5000,r=0.10, t=5

a) Compounded annually (m=1)

$FV = 5000(1 + 0.10)^5 = \$8052.55$

b) Compounded monthly (m=12)

$FV = 5000(1 + \frac{0.12}{12})^{60} = \$8226.54$

c) Compounded continuously

$FV = 5000*e^{0.10*5} = \$8243.61$

Part 4

Key takeaways

01 .Session Summary: Interest Rates Basics

Key Takeaways

1. Nominal vs. Effective Interest Rate

Nominal rate: Stated annual rate, does not account for compounding frequency.

Effective rate: Actual annual rate, reflects the true cost of borrowing or return on investment after compounding.

2. Effective Annual Rate (EAR)

Formula: $i_a = (1 + r/m)^m - 1$

r : Nominal annual rate

m : Number of compounding periods per year

Key rule: More frequent compounding → higher effective annual rate.

3. Continuous Compounding

Formula: $i_a = e^r - 1$

The theoretical limit of compounding (compounding infinitely often).

Used for advanced financial models and some investment calculations.

Next Session Preview

- Present Worth Analysis

Compare different project alternatives by converting all future cash flows to their present value equivalents.

- Economic Criteria for Project Evaluation

Learn core frameworks to decide which projects to accept/reject, based on financial goals.